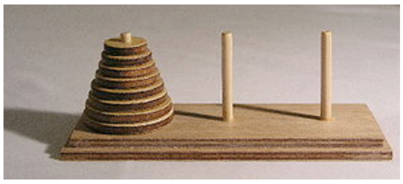
BACCALAUREAT- Session 2011-Section Européenne Anglais-- Epreuve de DNL Mathématiques-Série S



Document: Tower of Hanoi

The Tower of Hanoi is a mathematical game invented by the French mathematician Édouard Lucas in 1883. It consists of three rods, and a number of disks of different sizes, which can slide onto any rod. The puzzle starts with the disks in a stack in ascending order of size on one rod. The game's purpose is to move the n disks of the stack to the third rod, obeying the following rules:

1) Only one disk may be moved at a time.
2) Each move consists of taking the upper disk from one of the rods and sliding it onto another rod, on top of the other disks that may already be present on that rod.
3) No disk may be placed on top of a smaller disk.
4) The number of moves, denoted u_n, is to be kept to a minimum.

To solve the game with two disks (n=2), the number of moves u_2 to transfer the disk is $u_2=3$:

To solve the game with n+1 disks, assume that we know the number of moves u_n to slide a stack of n disks (represented by the triangle on the following diagram) onto the 2nd rod:

Beginning

…There's been u_n moves to transfer the stack of n disks onto the 2nd rod

Slide the (n+1)th disk onto the 3rd rod

Get the stack of (n+1) disk on the 3rd rod.

Thus the number of moves to transfer n+1 disks (represented by the triangle plus the additional disk) is $\mathbf{u_{n+1}=2u_n + 1}$.

From various sources

Questions

1. Use a diagram to show that the number of moves if n=1 is $u_1 = 1$, and if n=3 is $u_3=7$.
2. Check that $u_2=2u_1 + 1$ and $u_3=2u_2 + 1$.
3. Refer to the 2nd diagram: how many moves are there between the 3rd and the 4th drawing to slide the n disks of the stack onto the 3rd rod? Deduce why $u_{n+1}=2u_n + 1$.
4. Let (u_n) and (v_n) be the sequences defined for $n \geq 1$ by:

$$u_{n+1}=2u_n + 1 \text{ for } n \geq 2 \text{ and } u_1=1 \text{ and } v_n=u_n + 1 \text{ for } n \geq 1.$$

5. Give the definition of arithmetic and geometric sequences. Is (u_n) an arithmetic sequence? A geometric sequence?
6. Show that (v_n) is a geometric sequence of which you'll give the first term and the common ratio.
7. Give the expression of v_n in terms of n then deduce the expression of u_n in terms of n.
8. Compute u_{64}.