SOLVING SYSTEMS OF LINEAR EQUATIONS IN TWO VARIABLES

Lx

Definitions

The solution of a system of equations are the values of the variables that make both equations correct at the same time. (You can also say:"A solution of a system of a linear equation is an ordered pair that satisfies all equations in the system.")

A system of linear equations can have exactly one solution, no solution, or infinitely many solutions.

Solving a system by the addition method

$\begin{cases} 2x+5y=8 \\ -x+3y=7 \end{cases}$	Choose a variable to eliminate.
$\begin{cases} 2x+5y=8 \\ -x+3y=7 \quad \times \mathbf{2} \end{cases}$	Multiply one or both equations by an appropriate nonzero constant so that the sum of the coefficients of one of the variables is zero.
$\begin{cases} 2x+5y=8 \\ -2x+6y=14 \end{cases}$ $11y=22$	Add the two equations together to obtain an equation in one variable.
$\frac{11}{11}y=\frac{22}{11}$ $\boxed{y=2}$	Solve the equation in one variable.
$2x+5\times 2=8$ $2x+10=8$ $2x=-2$ $\boxed{x=-1}$	Substitute the value obtained into either of the original equations to solve for the other variable.

Solving a system by the substitution method

$\begin{cases} 2x+5y=8 \quad (1) \\ -x+3y=7 \quad (2) \end{cases}$ $(2) \Rightarrow \quad -x=7-3y$ $x=-7+3y$	Choose an equation and solve for one variable in terms of the other variable. (We could have solved for y, but we chose the easier case to avoid fractions.)
$(1) \Rightarrow \quad 2\times(-7+3y)+5y=8$	Substitute the expression into the other equation.
$-14+11y=8$ $11y=8+14$ $11y=22$ $\boxed{y=2}$	Solve the equation in one variable.
$(2) \Rightarrow \quad x=-7+3\times 2$ $\boxed{x=-1}$	Substitute the value found into one of the original equations to find the value of the remaining variable.

The ordered pair formed is the solution to the system.

We can use set notation to describe the solution: $\{(-1\,,2)\}$ (say "the set consisting of the ordered pair $(-1\,,2)$").

You can check the solution by substituting the pair of values <u>into both equations</u> of the original system.

Particular cases

systems with no solution

$$\begin{cases} 2x+y=7 \\ -2x-y=8 \end{cases}$$
$$0=15$$

"$0=15$" is a false statement, therefore the system is <u>inconsistent</u>, and there is no solution.

NB: If these equations were graphed, we would have two parallel lines.

systems with an infinite number of solutions

$$\begin{cases} 2x+y=7 \\ -2x-y=-7 \end{cases}$$
$$0=0$$

"$0=0$" is a true statement, therefore the system is <u>dependent</u>, and there are an infinite number of solutions.

NB: If these equations were graphed, we would have two coincident lines.