

MATHS BUSKING

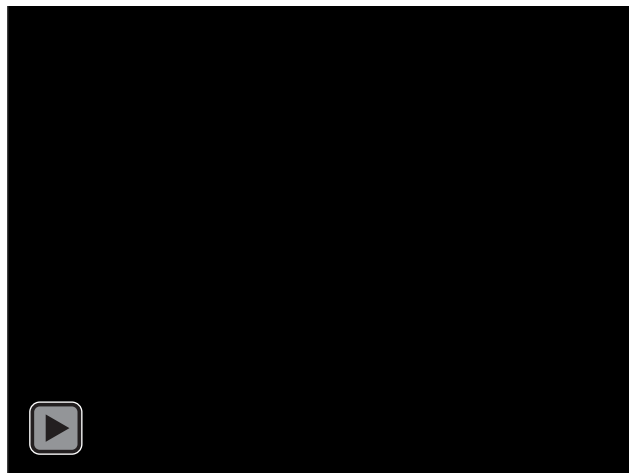


1 Vocabulary

Busker, trick, magic, card trick, to shuffle, to pick, to deal, pile, pack, mental calculation, digit, units digit, tens digit, to borrow, to subtract, times, to the power of, to cube, cubed, cubic root, favourite, estimate.

2 Watch the video and fill in the blanks

Maths busking is not a maths _____ outside. It's a whole new form of maths _____. Maths busking is real busking where maths is the gimmick, it's not just something else _____ with maths attached unto it, the mathematics is what the audience would genuinely find entertaining about the _____.



Maths busking makes people re-evaluate their _____ of mathematics and _____. Maths busking reaches a whole new _____ who would not otherwise engage with mathematics. Buskers need to engage with a passive and often indifferent audience, so what we're doing is taking the sensibilities in maths into busking and in using them to _____ mathematics. Maths busking is not just a new way to engage people with mathematics; it's also encouraging mathematicians to _____ better communication _____ and to think more about how they engage with the public.

Score: _____ out of 10

3 Cube root trick

To square a number, you multiply the number by itself. 4 squared is $4 \times 4 = 16$. You can write: $4^2 = 16$.

To cube a number, you multiply it by itself twice. 5 cubed is $5 \times 5 \times 5 = 125$. You can write: $5^3 = 125$.

The **cube root** (or **cubic root**) of a number is the value that when cubed gives the original number.

The cube root of 8 is 2, because when _____ is cubed you get _____. You can write $\sqrt[3]{8} = 2$.

What is the cube root of 125? _____ because when _____ is cubed you get _____.

Find the cube root of any whole number (less than a million) in your head!

First, let's find an estimate of the cubic root of a number.

Number	Number cubed
0	
10	
20	
30	
40	
50	
60	
70	
80	
90	
100	

Fill in the table.

Using the table, let's find an estimate of the cubic root of 79,507: it is between and .

Therefore you can guess the tens digit of the cube root.

You just have to memorize the cubes of the digits 1 through 9!

Now let's find a way to guess the units digit of the cubic root.

When you do a multiplication, how do you find the units digit of the result? Give an example.

Digits	Cubes	Last digit of the cubes
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		

Fill in the table.

The cube of a number ending in 1 ends also in .

The same happens for numbers ending in ,
 , .

The other pairs of numbers (2 and 8, and) can be easily memorized because their sum is always !

So, what is the cubic root of 79,507?



How to do the trick?

Ask your friend to choose any two-digit number in his head and to cube it with his calculator.

He then tells you the cube and keeps the original number secret.

For instance, let's say he picked 78, he computes $78 \times 78 \times 78$ with his calculator and tells you 474,552.

To find the original number, follow these two steps:

First, drop the last three digits and find in your head the largest cube less than or equal to 474: it is 7 because 7^3 is 343 and 8^3 is 512. So the tens digit of the cube root of 474,552 is **7**.

Now look at the last digit of the cubed number: 2, the same ending as 8^3 .

Thus the units digit of the cube root of 474,552 is **8**.

So the cube root of 474,552 is **78**.

Play the cube root trick with your partner and explain how you find the cube root of his number.

4 Number trick

Take any three-digit number in which the first and last digits differ by 2 or more (e.g. 387). Reverse the number (783), and subtract the smaller of the two numbers from the larger ($783 - 387 = 396$). Then reverse the result and add ($396 + 693 = 1089$).

Choose another three-digit number and write down your calculations. Try with another three-digit number.

The result is always 1089, no matter which three-digit number you start with!

Let's explain why the result is always 1089.

Let's write our three-digit number as ABC , where C is the number of units, B is the number of tens and A the number of hundreds. $ABC = 100A + 10B + C$.

When you reverse this number, you get CBA .

Let's say ABC is the larger of the two numbers. $ABC - CBA = (100A + 10B + C) - (100C + 10B + A)$.

Since $ABC > CBA$, A is greater than C . So to subtract the ones $C - A$, we need to borrow a ten.

So the number of ones is $C - A + 10$ and the number of tens is -1 .

We need to borrow a hundred. The number of hundreds is $A - C - 1$ and the number of tens is $10 - 1 = 9$.

Fill in the table below.

	Number of hundreds	Number of tens	Number of ones
ABC	A		C
– CBA	– C		
ABC – CBA	A – C		
ABC – CBA	A – C		C – A + 10
ABC – CBA	A – C – 1		C – A + 10
Reverse of (ABC – CBA)	C – A + 10		A – C – 1
Add last two numbers			9

Explain what happens when you choose a three-digit number in which the first and last digits differ by less than 2.

5 Mind reading

How to play the trick.

Have your spectators choose a small number and double it.

Add 8 to the number.

Divide the number by 2.

Subtract the original chosen number from the new number.

Convert this number to a corresponding letter: A = 1, B = 2, C = 3, D = 4, E = 5, F = 6.

Have your spectators think of a country that begins with this letter.

Have the spectator think of an animal that begins with the second letter of the chosen country.

Look at your spectators and say, "Oh, you know there are no elephants in Denmark!"

Work out the algebra to show that the final number is always 4.

Invent another trick based on the same principle where you end up with your favourite number.

6 Card trick

Watch the video and explain how the card trick is played.



The dealer starts with 27 cards and has a volunteer secretly choose one card, and asks the volunteer his favourite number between 1 and 27 (e.g. 8 in the video).

Without showing the dealer, the volunteer puts the card back anywhere into the pile of cards.

The dealer deals out three piles with nine cards in each pile and asks the volunteer which pile the card is in.

The volunteer tells the dealer and the dealer gathers up the cards.

The dealer repeats this two more times, and then is able to show the volunteer that the card he picked is the 8th card in the pile.

How does this trick work?

The maths behind this trick involves writing **numbers in base 3**.

The ternary numeral system (base 3) is a notation for numbers using the **powers of 3** rather than the powers of 10. Only the digits “0”, “1” and “2” are used to write numbers in base 3.

The number 19 writes as **201** in **base 3** because $19 = 2 \times 3^2 + 0 \times 3^1 + 1 \times 3^0$.

The number 27 writes as **1000** in **base 3** because $27 = 1 \times 3^3 + 0 \times 3^2 + 0 \times 3^1 + 0 \times 3^0$.

The number **2101** written in **base 3** is the number 64 because $2 \times 3^3 + 1 \times 3^2 + 0 \times 3^1 + 1 \times 3^0 = 64$

How do you write 7 in base 3?

How do you write 26 in base 3?

In what follows, the face of a card is the side that shows its value and suit; the back of the card is the side with the pattern on it; the position of the card always refers to piles of cards back up, face down.

Explanations when 8 is the favourite number chosen by the volunteer.

Subtract one to the favourite number and write it in base 3: 7 writes as 021 in base 3.

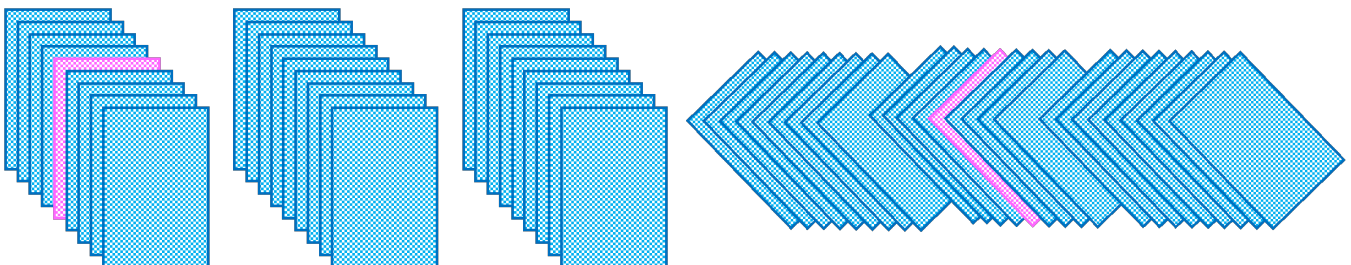
Reading this number in base 3 from right to left gives you the combination: middle (1) - bottom (2) – top (0) which corresponds to where to put the pile which contains the chosen card when you gather up the piles with the back of the cards up, face down.

First you deal out the cards in 3 piles of 9 cards (face down in your hand, dealt out face up for the spectator to see the cards and tell which pile contains the chosen card).

The chosen card is in one of the pile in position n with $1 \leq n \leq 9$ (cards in pile face down).

You can write $n = 3m + p$ with $0 \leq m \leq 2$ and $1 \leq p \leq 3$.

When you gather up the 3 piles (face down), you put this pile in the middle so there are 9 cards on top of this pile.

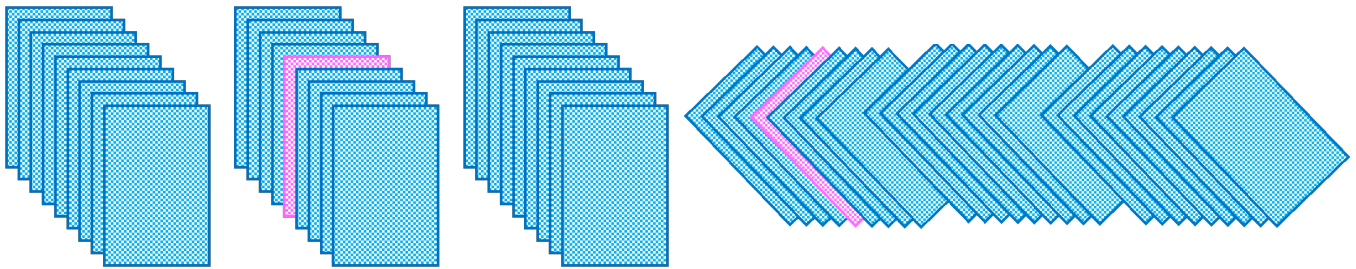


The chosen card is now in position $9 + n = 9 + 3m + p = 3(3 + m) + p$ in the pack of 27 cards (face down).

Again you deal out the cards in 3 piles the same way as earlier.

The chosen card is in position $3 + m + 1$ in its pile (in the 1st pile if $p = 1$, 2nd pile if $p = 2$, 3rd pile if $p = 3$).

When you gather up the 3 piles (face down), you put this pile at the bottom so there are 2 piles of 9 cards (18 cards) on top of this pile.

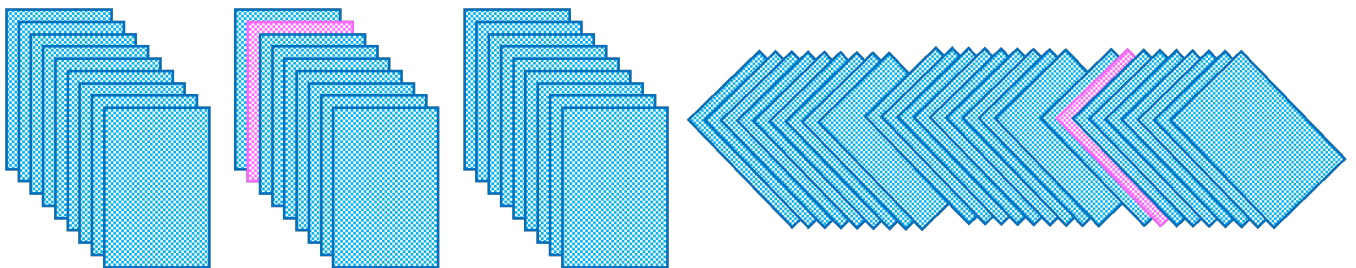


The chosen card is now in position $18 + 3 + (m + 1) = 21 + r = 3 \times 7 + r$ with $r = m + 1$, $1 \leq r \leq 3$ in the pack of 27 cards (face down).

For the last time, you deal out the cards in 3 piles (face up).

The chosen card is in position $7 + 1 = 8$ in its pile (in the 1st pile if $r = 1$, 2nd pile if $r = 2$, 3rd pile if $r = 3$).

You put this pile on top when you gather up the piles.



So the chosen card is indeed the 8th card of the pack!

Explanations when N is the favourite number chosen by the volunteer.

There are 27 cards so $1 \leq N \leq 27$.

$N - 1$ can be written in base 3 as: $N_3 N_2 N_1$ with $N - 1 = 9N_3 + 3N_2 + N_1$ and $0 \leq N_1 \leq 2$, $0 \leq N_2 \leq 2$, $0 \leq N_3 \leq 2$.

You deal out the cards in 3 piles of 9 cards.

The chosen card is in one of the three piles in position n with $1 \leq n \leq 9$ in this pile of 9 cards.

You can write $n = 3m + p$ with $0 \leq m \leq 2$ and $1 \leq p \leq 3$.

When you gather up the 3 piles, you put N_1 piles of 9 cards on top of this pile.

The chosen card is now in position $9N_1 + n = 9N_1 + 3m + p = 3(3N_1 + m) + p$ in the pack of 27 cards.

Again you deal out the cards in 3 piles of 9 cards.

The chosen card is in position $3N_1 + m + 1$ in its pile (1st pile if $p = 1$, 2nd pile if $p = 2$, 3rd pile if $p = 3$).

When you gather up the 3 piles, you put N_2 piles of 9 cards on top of this pile.

The chosen card is now in position $9N_2 + 3N_1 + (m + 1) = 3(3N_2 + N_1) + r$ with $r = m + 1$, $1 \leq r \leq 3$ in the pack of 27 cards.

You deal out the cards in 3 piles one last time.

The chosen card is now in position $3N_2 + N_1 + 1$ in its pile (1st pile if $r = 1$, 2nd pile if $r = 2$, 3rd pile if $r = 3$).

When you gather up the 3 piles, you put N_3 piles of 9 cards on top of this pile.

So the chosen card is indeed in position $9N_3 + 3N_2 + N_1 + 1 = N$ in the pack of 27 cards!

Explain how you play the trick for your favourite number between 1 and 27 and play the trick with your partner.

7 Homework

Choose one task from the following for your homework.

- ★ Write a one-page essay about how *maths busking* and *mathemagic* changed your opinion about maths.
- ★ Learn another card or number trick on the internet, show it to your classmates and explain the maths behind it.
- ★ Write an algorithm that gives the suitable top-middle-bottom combination for any given number in the three-pile card trick.
- ★ Write a proof similar to the one given here for a generalization of the game with n^m cards by dealing out the cards in n piles m times and using numbers written in base n (try the general case or choose a small value for n and m).

Name: