BACCALAUREAT Session 2011

Section Européenne Anglais-Série S

Epreuve de Mathématiques (DNL)

Document: Quaternion

In mathematics, the quaternion, denoted $\mathbb{H}$, is a hyper complex number system that extends the complex numbers. This extension is similar to the extension from real numbers to complex numbers.

Irish mathematician Sir William Rowan Hamilton in 1843 first described them. He knew that the complex numbers could be viewed as points in a plane, and he was looking for a way to do the same for points in space of which coordinates are a triple of numbers. But he had been stuck on the problem of division.

To overcome this problem, his idea was the use of a fourth dimension of space for the purpose of calculating with triples. The first dimension is a scalar (real part) and the three others are the components of a vector (imaginary part) in three dimensions.

A quaternion is therefore of the form: $\mathbb{H}$ = a.1 + b.i + c.j +d .k given a, b, c, d real numbers.

A striking feature of quaternion is that the product of quaternion is non-commutative, meaning that the product of quaternion depends on which factor is to the left of the multiplication sign and which factor is to the right. In the following table are given the rules of multiplication:

x	1	i	j	k
1	1	I	j	k
i	i	-1	k	-j
j	j	-k	-1	i
k	k	j	-i	-1

Quaternion is used in computer graphics, computer vision, robotics, control theory, signal processing, attitude control, physics, bioinformatics, molecular dynamics computer simulation and orbital mechanics. They also have received another boost from number theory because of their relation to quadratic forms.

From various sources

Questions

1. Where and when did Hamilton live?
2. Explain the principle of the extension from real numbers to complex numbers.
3. How can you represent real numbers graphically? What about complex numbers? So what was Hamilton's purpose?
4. What was the major problem encountered to describe quaternion? How was this problem surmounted?
5. Explain what "non-commutativity" is.
6. Explain what commutativity is and give an example.
7. Give three examples of fields in which quaternion are actually applied.