

QUADRATIC FUNCTIONS

parabolas

VOCABULARY

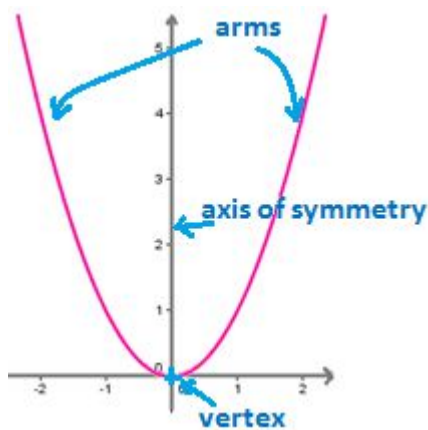
$f(x) = ax^2 + bx + c$ with a, b and c being real numbers, $a \neq 0$, is called a quadratic or a quadratic function.

$ax^2 + bx + c = 0$ with a, b and c being real numbers, $a \neq 0$, is called a quadratic equation.

The graph of a quadratic function is a parabola.

“Completing the square” is where we take an expression like this: $ax^2 + bx + c$, and turn it into this: $a(x + d)^2 + e$ or this: $a((x + d)^2 + e)$.

BASIC PARABOLA



$$y = x^2$$

The graph is symmetrical about the y -axis.

The minimum value of y occurs at the origin. This point is called the vertex of the parabola.

The arms of the parabola continue indefinitely.

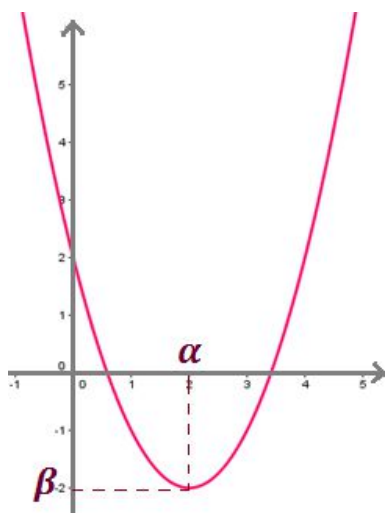
GENERAL CASE

$$\begin{aligned} f(x) &= ax^2 + bx + c \\ &= a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) \\ &= a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}\right) \quad \leftarrow \text{we complete the square} \\ &= a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right) \end{aligned}$$

$$\alpha = -\frac{b}{2a} \text{ and } \beta = -\frac{b^2 - 4ac}{4a}$$

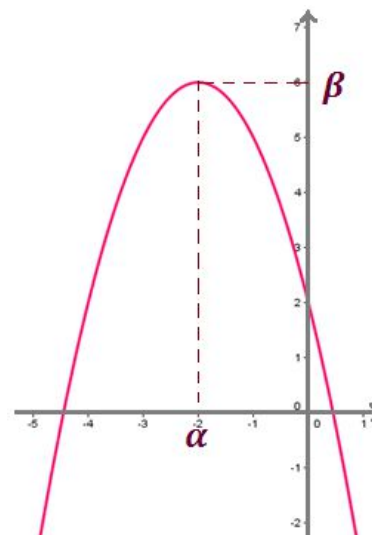
Note that $f(\alpha) = \beta$.

If $a > 0$



The curve opens up.

If $a < 0$



The curve opens down.

In both cases, the graph is symmetric to a vertical line through the vertex. This vertical line is called the axis of symmetry. Its equation is $x = \alpha$.