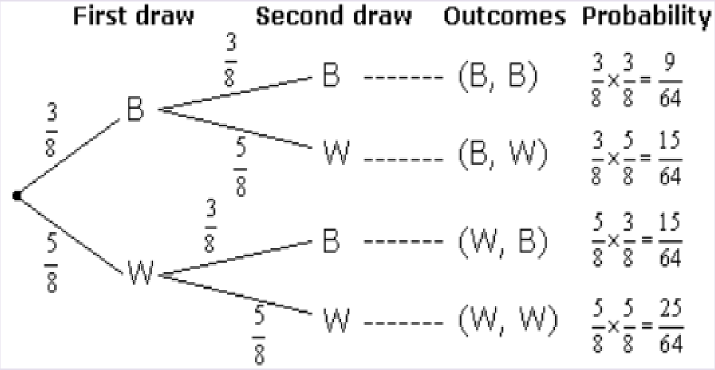
# PROBABILITY



## EXAMPLE

A bag contains 3 black balls and 5 white balls. Paul picks a ball at random from the bag and replaces it back in the bag. He mixes the balls in the bag and then picks another ball at random from the bag.




To calculate the probability of a branch, multiply probabilities along it.
Check that the probabilities in the last column add up to 1.

**"and" means "multiply"**

The probability of getting two black balls (that means getting a black ball from the first draw AND a black ball from the second draw) is:

$$P\big((B,B)\big) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$$

**"or" means "add"**

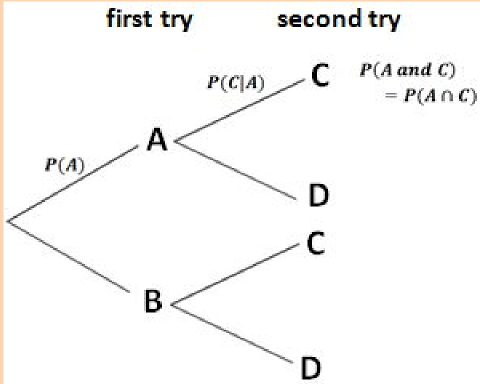
Let's calculate the probability of getting one and only one black ball.
Two outcomes satisfy this condition: (B,W) and (W,B). So:

$$P(getting\ exactly\ one\ black\ ball) = P((B,W)\,OR\,(W,B))$$
$$= \frac{3}{8} \times \frac{5}{8} + \frac{5}{8} \times \frac{3}{8}$$
$$= \frac{30}{64}$$

## CONDITIONAL PROBABILITY

A **conditional probability** is a probability of an event given another one.

$P(C|A)$ means "the probability of C given A".




$$\boxed{P(A \cap C) = P(A) \times P(C|A)}$$

(**multiplication rule**)

## DEPENDENT/INDEPENDENT EVENTS

Two events are **independent** if they have no influence on each other. In this case, the probability that they both occur is equal to the product of the probabilities of the two individual events: $P(A \cap B) = P(A) \times P(B)$.

## EXERCISE 1

Twin brothers, Ed and Jim, deliver the evening newspaper six nights a week. Ed delivers on two nights, chosen at random, and Jim on the other nights. They ride by a house on their bicycle and throw the newspaper onto the porch.

The probability that Ed hits the door is $\frac{3}{5}$ and the probability that Jim hits the door is $\frac{1}{10}$.

**1)** What is the probability that it is Jim's night and that the newspaper doesn't crash against the door?

**2)** What is the probability that one night the newspaper crashes against the door?

**3)** One night, while Mr Jones is watching TV before dinner, he hears a paper crashing against the door. He sighs to Mrs Jones: "It must be Ed's night ".
What is the probability that he is right?

*Nantes 2012*

## EXERCISE 2

In a game, a card is taken at random from a full deck of 52 playing cards.
It is then replaced, and a second card is taken.

Use a tree diagram to calculate the probability that:

**1) a)** both cards are diamonds
**b)** neither card is a diamond
**c)** one of the card is a diamond
**d)** at least one card is a diamond

**2)** Answer the same questions, assuming now that the first card is not replaced in the deck.

*Lille 2009*

## EXERCISE 3

A student randomly guesses at 10 multiple choice questions. Each question has four possible answers with only one being correct, and each is independent of every other question.
Find the probability that the student guesses exactly 5 correct.

**-> See : PROBABILITY Binomial distribution**