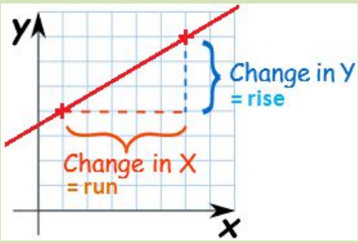
DIFFERENTIAL CALCULUS

Lesson

RECALL

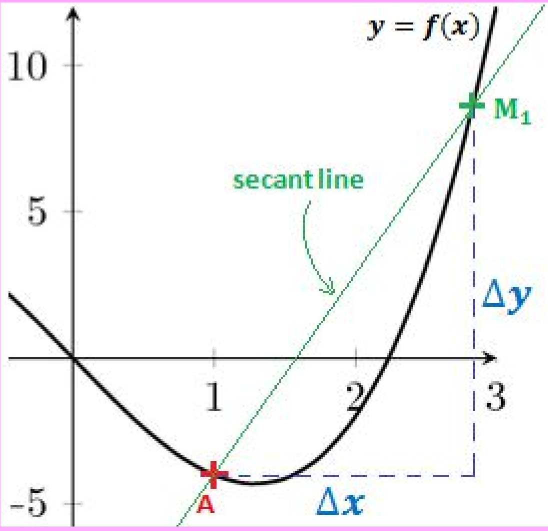


gradient (= slope) of a (straight) line

$$= \frac{\text{change in y}}{\text{change in x}}$$

$$= \frac{\text{rise}}{\text{run}}$$

SECANT LINE



Δx represents a small change in x, and Δy represents the corresponding change in y.

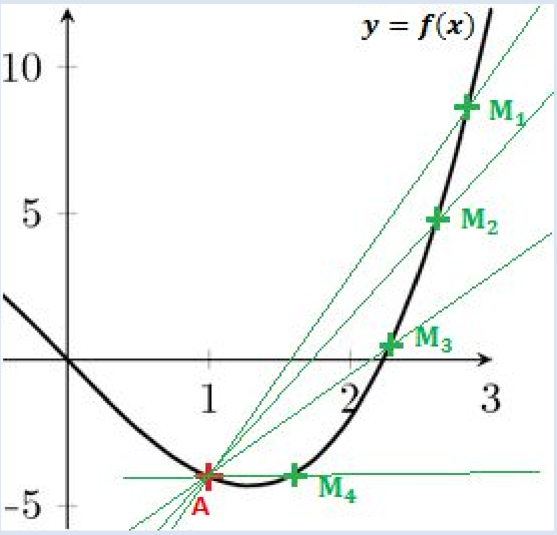
$$A\binom{x}{y} \text{ and } M_1\binom{x+\Delta x}{y+\Delta y}$$

or

$$A\binom{x}{f(x)} \text{ and } M_1\binom{x+\Delta x}{f(x+\Delta x)}$$

Thus the average slope of the secant line AM_1 is equal to $\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$.

TANGENT



As Δx becomes smaller and smaller (or "as Δx shrinks towards zero", or "as Δx heads towards zero"), this secant line approaches a line called the **tangent to the curve at A**.

The gradient of the tangent line is given by $\lim_{\Delta x \to o} \frac{f(x+\Delta x)-f(x)}{\Delta x}$.

If this limit exists, it is denoted by $\boldsymbol{\frac{d}{dx} f(x)}$, which is read "the derivative of $f(x)$" or "$d\ dx\ of\ f(x)$"
or by $\boldsymbol{f'(x)}$, which is read also "the derivative of $f(x)$" or "$f - dash\ of\ x$".

HISTORY

Calculus was discovered independently in the late 17th century by Newton and Leibniz.

Leibniz (1646 – 1716)

- German polymath and philosopher

- introduced the notation $\frac{dy}{dx}$, which refers to the instantaneous rate of change of y with respect of x.

Newton (1642 – 1726/27)

-English mathematician, astronomer and physicist

- described his version of differential calculus as 'the method of fluxions'

VOCABULARY

The process of finding a derivative is called "**differentiation**".
You **do** differentiation to **get** a derivative.
Functions which have derivative are called "**differentiable functions**".
differentiable **at** $x = 3$ / differentiable **on** a set
If the derivative at a point does not exist, you can say that "**the derivative is undefined**".

THE PRODUCT RULE

Let f, g be differentiable functions.
Then the derivative of their product is $f'g + g'f$

THE QUOTIENT RULE

Let f, g be differentiable functions.
Then the derivative of their quotient is $\frac{f'g-g'f}{g^2}$.