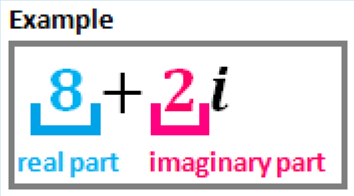
COMPLEX NUMBERS

Lesson

DEFINITION

A <u>complex number</u> is a number that can be written in the form $z = a + bi$, where a and b are real numbers, and i satisfies the equation $x^2 = -1$.



a is called the <u>real part</u> of z, and is denoted by $Re(z)$.
b is called the <u>imaginary part</u> of z, and is denoted by $Im(z)$.

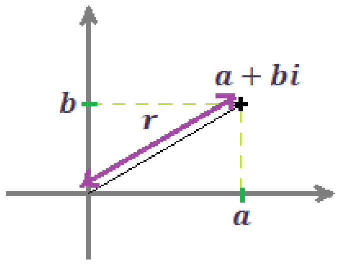
If $a = 0$, z is <u>purely imaginary</u>.
If $b = 0$, z is <u>purely real</u>.

CONJUGATE

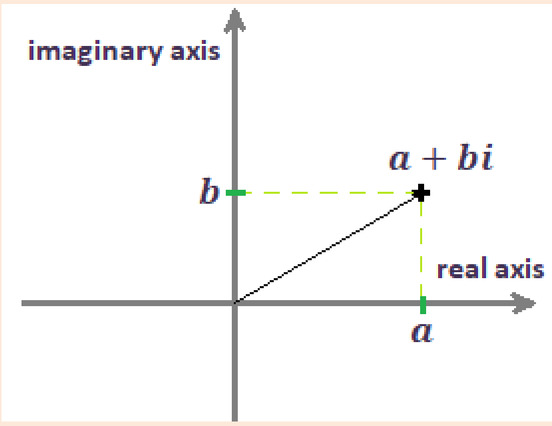
The <u>conjugate</u> of a complex number $a + bi$ is $a - bi$. It is often written with a bar over it: $\overline{a + bi} = a - bi$.

MODULUS (or magnitude)

Let $z = a + bi$.
The <u>modulus</u> of z is denoted $|z|$ or r and $|z| = r = \sqrt{a^2 + b^2}$.
It is the distance to the origin of the point representing z in the complex plane.

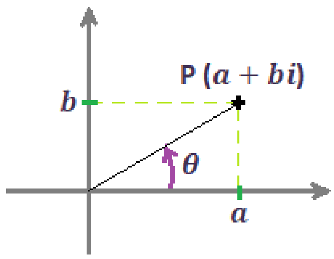


THE COMPLEX PLANE (or Argand plane)



ARGUMENT (or phase)

Let P be the point in the complex plane representing z.
The <u>argument</u> of z, denoted by $\arg z$ or θ, is the angle that line OP makes with the positive part of the real axis.



Note that θ is in radians.
-> <u>principal argument</u> if $\theta \in (-\pi, \pi]$

TRIGONOMETRIC FORM

$$\boxed{z = r(cos\theta + i\ sin\theta)}$$

The following equations relate a, b, r and θ:

$$a = r\cos\theta$$
$$b = r\sin\theta$$

POLAR FORM

$$\boxed{z = r\ e^{i\theta}}$$

Note that $\left|e^{i\theta}\right| = 1$.