SCHEDULING OF VEHICLES FROM A CENTRAL DEPOT TO A NUMBER OF DELIVERY POINTS

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The optimum routing of a fleet of trucks of varying capacities from a central depot to a number of delivery points may require a selection from a very large number of possible routes, if the number of delivery points is also large. This paper, after considering certain theoretical aspects of the problem, develops an iterative procedure that enables the rapid selection of an optimum or near-optimum route. It has been programmed for a digital computer but is also suitable for hand computation.

THE PAPER is concerned with the optimum routing of a fleet of trucks of varying capacities used for delivery from a central depot to a large number of delivery points. The merchandise is homogenous with respect to the unit of capacity. The shortest route between every two points in the system is given. It is desired to allocate loads to trucks in such a manner that all the merchandise is assigned and the total mileage covered is a minimum. The procedure given is simple but effective in producing a near-optimal solution and has been programmed for several digital computers. This truck dispatching (as a mathematical) problem was first formulated by DANTZIG AND RAMSER,[1,2] who obtained a method of solution. Multiple demand and multiple truck capacity were considered as a further formulation. The formulation in this paper is essentially similar, but a restriction was first imposed to meet a particular practical application for which the method was being developed. Their method has been developed and a new solution found.

FORMULATION

A NUMBER of trucks x_i of capacity C_i $(i=1\cdots n)$ are available and loads q_j are required to be delivered to points P_j $(j=1\cdots M)$ from a depot P_0. Given the distances $d_{y,z}$ between all such points it is required to minimize the total distance covered by the trucks.

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For convenience of computation C_i are ordered such that $C_{i-1}<C_i$ $(i=2\cdots n)$ and it is assumed that

$$C_1 \ll \sum_{j=1}^{j=m} q_j.$$

Since in the solution some trucks may only be partially loaded, x_1 needs to be sufficiently large to ensure that all loads are allocated. For purposes of computation it is therefore made infinite.

It might be noted that if $C_n \geqq \sum_{j=1}^{j=n} q_j$ then the problem becomes the traveling salesman problem.

COMMENTS ON THE DANTZIG AND RAMSER METHOD

THIS METHOD only considers the state $C_2=C_3=\cdots C_n=0$.

Due to the restriction that, in the first of N stages, only customers whose combined load does not exceed $C_1/2^{N-1}$ are permitted to be linked, points may be linked that are far apart, and may be virtually on opposite ends of a straight line through the depot. Although obviously long links may be excluded in the initial stages by 'rapid corrections,' when two points become linked in an 'aggregation' they remain aggregated.

As a result, this method tends to lay more emphasis on filling trucks to near capacity than on minimizing distance. The distance table in the Nth stage could require each cell to contain the shortest distance from the depot through 2^N points, i.e., the 'traveling salesman problem' must be solved; this can be extremely time consuming for only a few points, e.g., supposing after two stages 100 customers are aggregated into 25 groups of four customers each, then the mileage table for the next stage requires the solution of 300 traveling salesman problems each of nine points. These can of course be approximated too graphically.

A MODIFIED PROCEDURE BASED ON THE DANTZIG AND RAMSER METHOD

IF THE restriction is removed that in the rth stage aggregations, only customers whose combined load does not exceed $C_1/2^{N-r}$ may be joined, the 'traveling salesman problem' will still be encountered, but it is now permissible in each stage to join any two points whose combined load does not exceed C_1. This method has been found to give better results than the Dantzig and Ramser method in a number of cases tested. The relative merit of the two methods depends on the variability of the customer loads q_j. In the numerical example cited in reference 1 the original method gives 294 units, while this method gives 312 units.

This led the authors to seek another method of solution.

THEORETICAL ASPECTS OF THE PROBLEM

CONSIDER A feasible allocation of trucks to loads. In all cases each customer point will be linked to two other points, one or both of which may be the depot P_0. Consider the two points P_y and P_z. Let the two points linked to P_y be $P_{y\pm 1}$ and similarly for P_z. The effect of linking P_y and P_z will be calculated. It is assumed that P_y and P_z are on separate

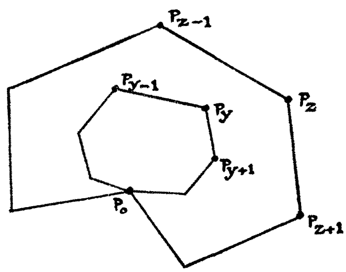

Figure 1

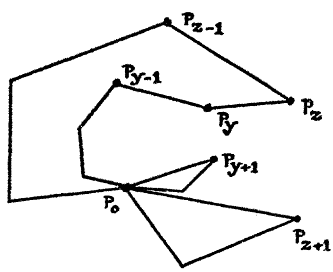

Figure 2

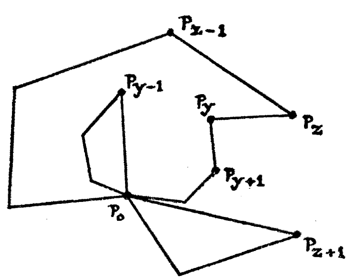

Figure 3

'runs' from P_0. If they are on the same 'run' the same considerations apply except that one of the four cases is not permissible.

Figure 1 shows the positions of P_y and P_z in the feasible allocation. Figures 2, 3, 4, 5 show the four possible decompositions of these runs caused by joining P_y and P_z. These consist of the severing of links $P_{y-1}\,P_y$ or $P_y\,P_{y+1}$ with the severing of links $P_{z-1}\,P_z$ or $P_z\,P_{z+1}$. The distances saved by each of these decompositions are as follows:

(2) $d_{y,y+1}-d_{0,y+1}+d_{z,z+1}-d_{0,z+1}-d_{y,z}$,

(3) $d_{y-1,y}-d_{0,y-1}+d_{z,z+1}-d_{0,z+1}-d_{y,z}$,

(4) $d_{y,y+1}-d_{0,y+1}+d_{z,z-1}-d_{0,z-1}-d_{y,z}$,

(5) $d_{y-1,y}-d_{0,y-1}+d_{z,z-1}-d_{0,z-1}-d_{y,z}$.

These four 'savings' are calculated for each pair of customers.

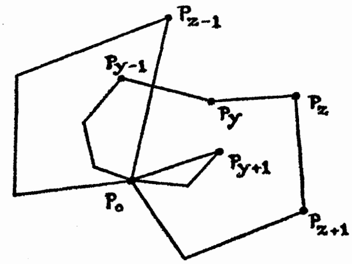

Figure 4

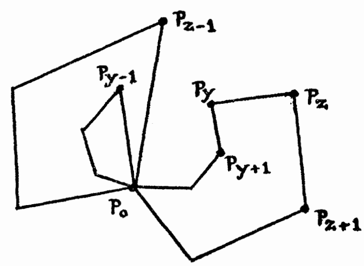

Figure 5

The maximum of these is selected that would, if linked, produce feasible routes consistent with truck availabilities and capacities. These two customers are now linked and the 'savings' recalculated. In linear programming terminology this method is equivalent to allotting two 'shadow costs' to a customer, the shadow cost for customer P_y being $d_{y-1,y}-d_{0,y-1}$ and $d_{y,y+1}-d_{0,y+1}$. The four 'evaluations' for cell $(y:z)$ are then enumerated above. When a link is severed, the appropriate 'shadow cost' is reduced by the value of the cell causing the severance. Since this cell value is a maximum value, whenever a point is linked to two others (not P_0) all its cell values become negative, and this point will not be considered again for linking.

As a result of this, the only links that will be severed will be those of points linked to P_0, if these were involved in the linking of cell $(y{:}z)$, the saving would be $d_{0,y}+d_{0z}-d_{yz}$.

TABLE I

Q	P_0												
1200	2 9	P_1											
1700	2 14	18 5	P_2										
1500	2 21	18 12	28 7	P_3									
1400	2 23	10 22	20 17	34 10	P_4								
1700	2 22	10 21	20 16	22 21	26 19	P_5							
1400	2 25	10 24	16 23	16 30	20 28	38 9	P_6						
1200	2 32	10 31	20 26	26 27	30 25	44 10	50 7	P_7					
1900	2 36	10 35	20 30	20 37	24 35	42 16	50 11	58 10	P_8				
1800	2 38	10 37	16 36	16 43	20 41	38 22	50 13	54 16	68 6	P_9			
1600	2 42	10 41	20 36	32 31	36 29	44 20	50 17	64 10	72 6	68 12	P_{10}		
1700	2 50	10 49	20 44	34 37	42 31	44 28	50 25	64 18	72 14	76 12	84 8	P_{11}	
1100	2 52	10 51	20 46	34 39	46 29	44 30	50 27	64 20	72 16	70 20	84 10	92 10	P_{12}

COMPUTATIONAL PROCEDURE

It is assumed that the values of q_j are such that an initial allocation of one vehicle to each customer is possible. If this is not true it is assumed that an allocation can be made by splitting a load into two (or more) full truck-loads of the highest capacities available and only considering the remainder of that load, an amount less than a truckload of the highest capacity. E.g., $q_j = 1700$ gallons and the available trucks are one of 700 gallons, two of 600 gallons, three of 500 gallons then the 700 gallon and 600 gallon trucks would be allocated to the customer and 400 gallons used as the value of q_j with an availability now of three 500 gallon trucks. Hence $q_j \leqq C_n$ $(j=1\cdots M)$.

For hand computation it is recommended that half matrix be used. The numerical example used here is the same as that used in reference 1. In Table I the entries in the lower right-hand corner of each cell $(y{:}z)$ are the appropriate distances $d(P_y{:}P_z)$ by the shortest practicable route. The entries in the lower left-hand corner of each cell are the 'savings.' For cell $(y{:}z)$ with $y,z \geqq 1$ and the $y \neq z$ this value is $d_{0,y}+d_{0,z}-d_{y,z}$. A column vector $Q=(Q_1\cdots Q_M)$ is added on the left-hand side of the matrix. Initially this consists of the loads q_j required by customer P_j $(j=1\cdots M)$. The entry in the middle of cell $(y{:}z)$ is $t_{y,z}=1$ if the two customers P_y and P_z are linked on a truck's route; otherwise $t_{y,z}=0$ for $y,z>0$. If a customer

TABLE II

Trucks	Up to 4000 gal	Over 4000 gal	Over 5000 gal	Over 6000 gal
Available	∞	7	4	0
Allocated	12	0	0	0

is served exclusively by a truck $t_{y,0}=2$. The following relation always exists:

$$\sum_{z=0}^{y-1} t_{y,z} \sum_{z=y+1}^{z=M} t_{y,z} = 2 \cdots (A).$$

The initial basic solution is now entered as $t_{y,0}=2$ $(y=1\cdots M)$.

Table II is a table showing the number of available trucks above each capacity level and the number of trucks already allocated.

In the numerical example shown, it is assumed that there is an unlimited supply of trucks of capacity 4000 gallons, 3 trucks of capacity 5000 gallons, and 4 trucks of capacity 6000 gallons.

Tables I and II show the initial feasible solution.

The rows and columns of the half matrix are searched for the maximum 'saving,' subject to the conditions that if this occurs in cell $(y{:}z)$:

(I) $T_{y,0}$ and $t_{z,0}$ must be greater than zero.

(II) P_y and P_z are not already allocated on the same truck run.

(III) Amending Table II by removing the trucks allocated to leads Q_y and Q_z and adding a truck to cover the load Q_y+Q_z does not cause the trucks allocated to exceed the trucks available in any column of Table II.

TABLE III

Q	P_0												
1200	2	P_1											
1700	2	18	P_2										
2900	1	18	28[a]	P_3									
2900	1	10	20	34^{1}	P_4								
1700	2	10	20	22	26	P_5							
5100	1	10	16	16	20	38	P_6						
5600	1	10	20	26	30	44	50	P_7					
—		10	20	20	24	42	50^{1}	58	P_8				
5100	1	10	16	16	20	38	50	54	68^{1}	P_9			
5600	1	10	20	32	36	44	50	64	72	68	P_{10}		
—		10	20	34	42	44	50	64^{1}	72	76	84	P_{11}	
—		10	20	34	46	44	50	64	72	70	84^{1}	92^{1}	P_{12}

[a] indicates maximum saving satisfying all conditions (1), (11), and (111) for use in next iteration.

TABLE IV

Trucks	Up to 4000 gal	Over 4000 gal	Over 5000 gal	Over 6000 gal
Available	∞	7	4	0
Allocated	4	2	2	0

If these conditions hold, $t_{y,z}$ is made equal to 1 and other values of $t_{i,j}$ amended subject to relation (A). The vector Q is amended by firstly

TABLE V

Q	P_0												
5800	1	P_1											
—		1	P_2										
—			1	P_3									
5800	1			1	P_4								
1700	2					P_5							
5100	1						P_6						
5600	1							P_7					
—							1		P_8				
5100	1								1	P_9			
5600	1										P_{10}		
—								1				P_{11}	
—											1	1	P_{12}

TABLE VI

Trucks	Up to 4000 gal	Over 4000 gal	Over 5000 gal	Over 6000 gal
Available	∞	7	4	0
Allocated	1	3	3	0

making all Q_j zero where $t_{j,0}$ is zero and making Q_j equal to the total load on the 'run' for all other j. This completes the first iteration.

If there are two or more equal maxima in the seach it is suggested that one of these be selected randomly. The procedure is repeated until no more links are possible. Tables III and IV show an intermediate stage in the computation. Tables V and VI show the final solution. The routes are as follows: $P_0P_1P_2P_3P_4P_0$, $P_0P_5P_0$, $P_0P_6P_8P_9P_0$, $P_0P_{10}P_{12}P_{11}P_7P_0$, giving a total distance of 290 units, believed by Dantzig and Ramser to be optimum.

Although the improvement in this example is slight, in an example with 30 customers an improvement of 17 per cent on the earlier method was obtained. The results of this example with the initial mileage matrix are given in the appendices to this paper.

While the solution does give the order of visiting the customers it may be beneficial to solve the traveling salesmen problem for each truck in the final allocation to obtain the true optimum order of visiting.

Practical limitations such as certain customers only accepting certain truck sizes or types and other priority treatments can be incorporated into the computation without much difficulty. Details of some of these restrictions together with computational methods for a digital computer will be found in a Case Study which will be published shortly.

Appendix I

MIDLANDS II

Proposed Runs for December 4, 1961, using Dantzig's Method

Code No.	Society	T	Cwt	Load T	Load Cwt	Mileage
1	Stoke	1	4			
2	Burslem	1	14	2	18	82
3	Barrow on Soar		11			
4	Mount Sorrel		15			
5	Fleckney		11			
6	Stoughton		1			
7	Huncote		3			
8	Dudley	1	9			
9	Sapcote		6			
10	Tenacres	1	5	5	1	238
11	Whetstone		6			
12	Tamworth	1	5			
13	Enderby		2			
14	Nuneaton	1	8			
15	Broughton Astley		8			
16	Cosby		10	3	19	201
17	Silverdale		18			
18	Walsall	2	5			
19	Birmingham	1	13			
20	Stafford		17			
21	Market Drayton		9	6	2	183
22	Shepshed		16			
23	Coalville	1	15			
24	Loughborough		5			
25	Wolverhampton	3	0	5	16	201
26	Coventry	4				
27	Lockhurst Lane	1	19	5	19	197
28	Melton Mowbray	4	15	4	15	186
29	Leicester	4	10	4	10	178
30	Oakengates	6	3	6	3	136
19	Birmingham	7	0	7	0	164
	Total					1,766

TABLE VII
MILEAGE TABLE

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0	Newton Heath																															
1	Stoke	41																														
2	Burslem	38	3																													
3	Barrow	80	54	56																												
4	Mt. Sorrel	80	54	56	3																											
5	Fleckney	97	64	67	19	16																										
6	Stoughton	92	59	62	13	10	7																									
7	Huncote	96	56	59	16	14	11	10																								
8	Dudley	78	39	41	54	54	53	57	48																							
9	Sapcote	98	59	62	20	17	12	13	4	39																						
10	Tenacres	87	52	50	47	46	46	42	35	12	33																					
11	Whetstone	95	58	61	15	12	8	8	3	45	6	39																				
12	Tamworth	77	38	41	30	29	34	32	25	24	21	18	28																			
13	Enderby	93	55	58	15	12	10	14	3	47	7	39	3	26																		
14	Nuneaton	91	52	53	25	22	24	19	12	30	9	24	15	14	14																	
15	Broughton	98	58	61	19	16	10	10	4	42	3	36	4	24	6	12																
16	Cosby	96	59	62	17	14	8	8	4	44	5	38	2	26	4	14	2															

17	Silverdale	40	45	5	60	61	71	65	63	40	66	49	65	40	62	54	65	66													
18	Walsall	73	34	37	46	46	50	46	39	8	39	12	43	16	41	28	42	44	36												
19	Birmingham	82	48	46	44	44	45	42	33	9	31	4	36	18	36	21	34	35	45	9											
20	Stafford	55	16	19	54	54	58	55	48	22	45	30	53	24	51	38	48	50	18	22	31										
21	Market Drayton	52	16	17	68	68	77	72	69	36	65	46	71	44	68	57	67	69	14	36	45	21									
22	Shepshed	76	46	49	8	9	19	15	18	44	22	40	16	20	17	24	20	18	52	37	35	45	59								
23	Coalville	76	44	46	11	11	20	15	15	42	19	36	18	18	14	18	20	18	47	35	33	39	57	6							
24	Loughbro'	76	50	53	4	4	20	14	18	50	21	43	17	25	16	28	20	19	57	41	40	50	64	5	10						
25	Wolverhampton	72	33	34	53	54	57	55	47	6	45	15	49	22	49	34	46	48	34	6	15	16	30	47	42	50					
26	Coventry	98	58	61	33	30	27	30	21	27	15	18	19	19	21	8	17	19	60	26	18	44	61	34	28	35	32				
27	Lockhurst L.	98	58	61	32	29	26	29	20	28	15	20	18	19	20	7	16	18	60	26	19	44	61	34	28	34	34	3			
28	Melton M.	93	66	68	14	12	23	18	22	65	25	54	20	41	20	32	24	22	72	57	54	61	79	20	26	15	64	39	39		
29	Leicester	89	55	58	10	9	8	5	7	48	10	39	5	29	5	18	9	7	62	44	39	51	69	15	12	11	52	24	23	15	
30	Oakengates	68	32	33	64	64	67	66	57	22	55	38	63	34	60	47	58	60	32	26	33	21	18	66	53	60	18	51	52	76	65

Appendix II

Midlands II

Proposed Runs for December 4, 1961, using proposed method with 7-ton Capacity.

Code No.	Society	T	Cwt	Load T	Load Cwt	Mileage
2	Burslem	1	14			
1	Stoke	1	4			
23	Coalville	1	15			
22	Shepshed	0	16	5	9	167
12	Tamworth	1	5			
14	Nuneaton	1	8			
10	Tenacres	1	5			
19	Birmingham	1	13			
8	Dudley	1	9	7	0	206
17	Silverdale	0	18			
27	Lockhurst Lane	1	19			
26	Coventry	4	0	6	17	201
20	Stafford	0	17			
25	Wolverhampton	3	0			
18	Walsall	2	5	6	2	150
24	Loughborough	0	5			
4	Mount Sorrel	0	15			
28	Melton Mowbray	4	15			
3	Barrow on Soar	0	11	6	6	186
6	Stoughton	0	1			
5	Fleckney	0	11			
11	Whetstone	0	6			
16	Cosby	0	10			
15	Broughton Astley	0	8			
9	Sapcote	0	6			
7	Huncote	0	3			
13	Enderby	0	2			
29	Leicester	4	10	6	17	215
30	Oakengates	6	3			
21	Market Drayton	0	9	6	12	138
19	Birmingham	7	0	7	0	164
			Total			1,427

ACKNOWLEDGMENT

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