PROBLEM.

Given the number of times in which an unknown event has happened and failed: *Required* the chance that the probability of its happening in a single trial lies somewhere between any two degrees of probability that can be named.

SECTION I.

DEFINITION 1. Several events are *inconsistent*, when if one of them happens, none of the rest can.

2. Two events are *contrary* when one, or other of them must; and both together cannot happen.

3. An event is said to *fail*, when it cannot happen; or, which comes to the same thing, when its contrary has happened.

4. An event is said to be determined when it has either happened or failed.

5. The *probability of any event* is the ratio between the value at which an expectation depending on the happening of the event ought to be computed, and the value of the thing expected upon it's happening.

6. By *chance* I mean the same as probability.

7. Events are independent when the happening of any one of them does neither increase nor abate the probability of the rest.

PROP. 1.

When several events are inconsistent the probability of the happening of one or other of them is the sum of the probabilities of each of them.

Suppose

Suppofe there be three fuch events, and which ever of them happens I am to receive N, and that the probability of the 1ft, 2d, and 3d are refpectively $\frac{a}{N}$, $\frac{b}{N}$, $\frac{c}{N}$. Then (by the definition of probability) the value of my expectation from the 1ft will be a, from the 2d b, and from the 3d c. Wherefore the value of my expectations from all three will be $a + b + c$. But the fum of my expectations from all three is in this cafe an expectation of receiving N upon the happening of one or other of them. Wherefore (by definition 5) the probability of one or other of them is $\frac{a+b+c}{N}$ or $\frac{a}{N} + \frac{b}{N} + \frac{c}{N}$. The fum of the probabilities of each of them.

Corollary. If it be certain that one or other of the three events muft happen, then $a + b + c = N$. For in this cafe all the expectations together amounting to a certain expectation of receiving N, their values together muft be equal to N. And from hence it is plain that the probability of an event added to the probability of its failure (or of its contrary) is the ratio of equality. For thefe are two inconfiftent events, one of which neceffarily happens. Wherefore if the probability of an event is $\frac{P}{N}$ that of it's failure will be $\frac{N-P}{N}$.

PROP. 2.

If a perfon has an expectation depending on the happening of an event, the probability of the event is to the probability of its failure as his lofs if it fails to his gain if it happens.

Suppofe a perfon has an expectation of receiving N, depending on an event the probability of which is is $\frac{P}{N}$. Then (by definition 5) the value of his expectation is P, and therefore if the event fail, he loſes that which in value is P; and if it happens he receives N, but his expectation ceaſes. His gain therefore is N—P. Likewiſe ſince the probability of the event is $\frac{P}{N}$, that of its failure (by corollary prop. 1) is $\frac{N-P}{N}$. But $\frac{P}{N}$ is to $\frac{N-P}{N}$ as P is to N—P, i. e. the probability of the event is to the probability of it's failure, as his loſs if it fails to his gain if it happens.

PROP. 3.

The probability that two ſubſequent events will both happen is a ratio compounded of the probability of the 1ſt, and the probability of the 2d on ſuppoſition the 1ſt happens.

Suppoſe that, if both events happen, I am to receive N, that the probability both will happen is $\frac{P}{N}$, that the 1ſt will is $\frac{a}{N}$ (and conſequently that the 1ſt will not is $\frac{N-a}{N}$) and that the 2d will happen upon ſuppoſition the 1ſt does is $\frac{b}{N}$. Then (by definition 5) P will be the value of my expectation, which will become b if the 1ſt happens. Conſequently if the 1ſt happens, my gain by it is b—P, and if it fails my loſs is P. Wherefore, by the foregoing propoſition, $\frac{a}{N}$ is to $\frac{N-a}{N}$, i. e. a is to N—a as P is to b—P. Wherefore (componendo inverſè) a is to N as P is to b. But the ratio of P to N is compounded of the ratio of P to b, and that of b to N. Wherefore the

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fame ratio of P to N is compounded of the ratio of a to N and that of b to N, i. e. the probability that the two fubfequent events will both happen is compounded of the probability of the 1ft and the probability of the 2d on fuppofition the 1ft happens.

Corollary. Hence if of two fubfequent events the probability of the 1ft be $\frac{a}{N}$, and the probability of both together be $\frac{P}{N}$, then the probability of the 2d on fuppofition the 1ft happens is $\frac{P}{a}$.

PROP. 4.

If there be two fubfequent events to be determined every day, and each day the probability of the 2d is $\frac{b}{N}$ and the probability of both $\frac{P}{N}$, and I am to receive N if both the events happen the 1ft day on which the 2d does; I fay, according to thefe conditions, the probability of my obtaining N is $\frac{P}{b}$. For if not, let the probability of my obtaining N be $\frac{x}{N}$ and let y be to x as N—b to N. Then fince $\frac{x}{N}$ is the probability of my obtaining N (by definition 1) x is the value of my expectation. And again, becaufe according to the foregoing conditions the 1ft day I have an expectation of obtaining N depending on the happening of both the events together, the probability of which is $\frac{P}{N}$, the value of this expectation is P. Likewife, if this coincident fhould not happen I have an expectation of being reinftated in my former circumftances, i. e. of receiving that which in value is x depending

pending on the failure of the 2d event the probability of which (by cor. prop. 1) is $\frac{N-b}{N}$ or $\frac{y}{x}$, becaufe y is to x as $N-b$ to N. Wherefore fince x is the thing expected and $\frac{y}{x}$ the probability of obtaining it, the value of this expectation is y. But thefe two laft expectations together are evidently the fame with my original expectation, the value of which is x, and therefore $P+y=x$. But y is to x as $N-b$ is to N. Wherefore x is to P as N is to b, and $\frac{x}{N}$ (the probability of my obtaining N) is $\frac{P}{b}$.

Cor. Suppofe after the expectation given me in the foregoing propofition, and before it is at all known whether the 1ft event has happened or not, I fhould find that the 2d event has happened; from hence I can only infer that the event is determined on which my expectation depended, and have no reafon to efteem the value of my expectation either greater or lefs than it was before. For if I have reafon to think it lefs, it would be reafonable for me to give fomething to be reinftated in my former circumftances, and this over and over again as often as I fhould be informed that the 2d event had happened, which is evidently abfurd. And the like abfurdity plainly follows if you fay I ought to fet a greater value on my expectation than before, for then it would be reafonable for me to refufe fomething if offered me upon condition I would relinquifh it, and be reinftated in my former circumftances; and this likewife over and over again as often as (nothing being known concerning the 1ft event) it fhould appear that the 2d had happened. Notwithftanding therefore this difcovery that the 2d event

event has happened, my expectation ought to be esteemed the same in value as before, i. e. x, and consequently the probability of my obtaining N is (by definition 5) still $\frac{x}{N}$ or $\frac{P}{b}$*. But after this discovery the probability of my obtaining N is the probability that the 1st of two subsequent events has happened upon the supposition that the 2d has, whose probabilities were as before specified. But the probability that an event has happened is the same as the probability I have to guess right if I guess it has happened. Wherefore the following proposition is evident.

PROP. 5.

If there be two subsequent events, the probability of the 2d $\frac{b}{N}$ and the probability of both together $\frac{P}{N}$, and it being 1st discovered that the 2d event has happened, from hence I guess that the 1st event has also happened, the probability I am in the right is $\frac{P}{b}$†.

PROP.

* What is here said may perhaps be a little illustrated by considering that all that can be lost by the happening of the 2d event is the chance I should have had of being reinstated in my former circumstances, if the event on which my expectation depended had been determined in the manner expressed in the proposition. But this chance is always as much *against* me as it is *for* me. If the 1st event happens, it is *against* me, and equal to the chance for the 2d event's failing. If the 1st event does not happen, it is *for* me, and equal also to the chance for the 2d event's failing. The loss of it, therefore, can be no disadvantage.

† What is proved by Mr. Bayes in this and the preceding proposition is the same with the answer to the following question. What is the probability that a certain event, when it happens, will

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PROP. 6.

The probability that ſeveral independent events ſhall all happen is a ratio compounded of the probabilities of each.

For from the nature of independent events, the probability that any one happens is not altered by the happening or failing of any of the reſt, and conſequently the probability that the 2d event happens on ſuppoſition the 1ſt does is the ſame with its original probability; but the probability that any two events happen is a ratio compounded of the probability of the 1ſt event, and the probability of the 2d on ſuppoſition the 1ſt happens by prop. 3. Wherefore the probability that any two independent events both happen is a ratio compounded of the probability of the 1ſt and the probability of the 2d. And in like manner conſidering the 1ſt and 2d event together as one event; the probability that three independent events all happen is a ratio compounded of the probability that the two 1ſt both happen and the probability of the 3d. And thus you

be accompanied with another to be determined at the ſame time? In this caſe, as one of the events is given, nothing can be due for the expectation of it; and, conſequently, the value of an expectation depending on the happening of both events muſt be the ſame with the value of an expectation depending on the happening of one of them. In other words; the probability that, when one of two events happens, the other will, is the ſame with the probability of this other. Call x then the probability of this other, and if $\frac{b}{N}$ be the probability of the given event, and $\frac{p}{N}$ the probability of both, becauſe $\frac{p}{N} = \frac{b}{N} \times x$, $x = \frac{p}{b} =$ the probability mentioned in theſe propoſitions.

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may proceed if there be ever so many such events; from whence the proposition is manifest.

Cor. 1. If there be several independent events, the probability that the 1st happens the 2d fails, the 3d fails and the 4th happens, &c. is a ratio compounded of the probability of the 1st, and the probability of the failure of the 2d, and the probability of the failure of the 3d, and the probability of the 4th, &c. For the failure of an event may always be considered as the happening of its contrary.

Cor. 2. If there be several independent events, and the probability of each one be a, and that of its failing be b, the probability that the 1st happens and the 2d fails, and the 3d fails and the 4th happens, &c. will be $abba$, &c. For, according to the algebraic way of notation, if a denote any ratio and b another, $abba$ denotes the ratio compounded of the ratios a, b, b, a. This corollary therefore is only a particular case of the foregoing.

Definition. If in consequence of certain data there arises a probability that a certain event should happen, its happening or failing, in consequence of these data, I call it's happening or failing in the 1st trial. And if the same data be again repeated, the happening or failing of the event in consequence of them I call its happening or failing in the 2d trial; and so on as often as the same data are repeated. And hence it is manifest that the happening or failing of the same event in so many diffe- trials, is in reality the happening or failing of so many distinct independent events exactly similar to each other.

 PROP.

PROP. 7.

If the probability of an event be a, and that of its failure be b in each single trial, the probability of its happening p times, and failing q times in $p+q$ trials is E $a^p b^q$ if E be the coefficient of the term in which occurs $a^p b^q$ when the binomial $\overline{a+b}|^{p+q}$ is expanded.

For the happening or failing of an event in different trials are so many independent events. Wherefore (by cor. 2. prop. 6.) the probability that the event happens the 1st trial, fails the 2d and 3d, and happens the 4th, fails the 5th, &c. (thus happening and failing till the number of times it happens be p and the number it fails be q) is $abbab$ &c. till the number of a's be p and the number of b's be q, that is; 'tis $a^p b^q$. In like manner if you consider the event as happening p times and failing q times in any other particular order, the probability for it is $a^p b^q$; but the number of different orders according to which an event may happen or fail, so as in all to happen p times and fail q, in $p+q$ trials is equal to the number of permutations that $aaaa\ bbb$ admit of when the number of a's is p, and the number of b's is q. And this number is equal to E, the coefficient of the term in which occurs $a^p b^q$ when $\overline{a+b}|^{p+q}$ is expanded. The event therefore may happen p times and fail q in $p+q$ trials E different ways and no more, and its happening and failing these several different ways are so many inconsistent events, the probability for each of which is $a^p b^q$, and therefore by

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