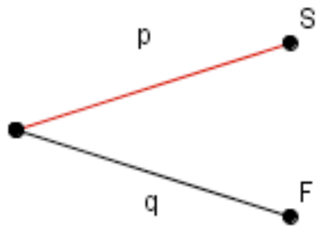
Binomial Distribution

Task 1 : Write a presentation about the Bernoulli family. Focus on James Bernoulli.

I. Bernoulli trials

A **Bernoulli trial** is an experiment whose outcome is random and can be either of **two possible outcomes**, ……………..and ……………….



Let **p** denote the probability of success in a **Bernoulli trial**, and so the probability of failure is

q =

Let X the random variable such that X=1 in case of Success and X=0 else (Failure).

The Bernoulli distribution is given by :

x_i	1	0
$P(X=x_i)$	p	

Note : The expectation is ***E(x) = p*** *and the variance is* ***V(x) = pq***

Task 2

A die is rolled. We consider the event "Obtaining 6". State the probability distribution of this experiment.

II. Bernoulli scheme

A **Bernoulli scheme** is a sequence of ***n*** independent identically distributed Bernoulli trials.

A result is a list of ***n*** outcomes, example: (S, S, F, F, F, …, F)

III. Binomial distribution

We consider the random variable Y mapping the **number of successes *k*** ($0 \le k \le n$) onto each outcome.

Examples : $(\underbrace{S, S, S, \ldots, S}_{\text{k successes}}, \underbrace{F, F, F, \ldots, F}_{\text{(n-k) failures}})$ is a list of k successes.

(S, S, S, ….,S, S, S, S, …, S) is a list of n successes
(F, F, F, ….,F, F, F, F, …, F) is a list of 0 successes

The probability of getting a list of k successes is :

$$P(Y = k) = n_k\, p^k\, q^{n-k}$$

given n_k : number of lists of k successes

(Admitted)

The probability distribution of Y is a **binomial distribution** with *n* trials and probability *p* of success. It is denoted ***B (n, p).***

k	0	1	2	…	…	n
P(y=k)		$n_1\, p^1\, q^{n-1}$				

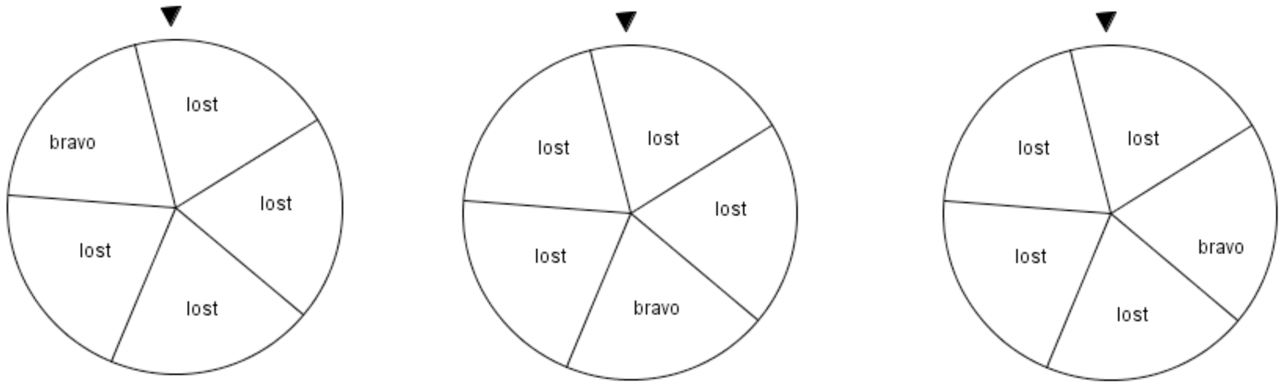
Note (TS):

$$n_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$ *(binomial coefficient ; read "n choose k")*

The expectation is **E(Y) =np** and the variance is **V(Y) =npq**

Task 3

A lottery consists of spinning three wheels as shown below:



The player wins on the lottery if at least one wheel stops on “**bravo**”.

- Verify that this experiment is a binomial distribution.
- Determine the probability to win this game.

Task 4

A student answers at random the 10 questions of a multiple choice question paper. Each question has 4 answers and only one is correct. The questions are independent.
Find the expectation of the number of good answers.

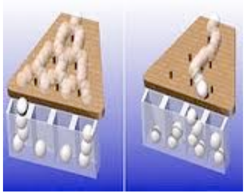
Task 5

Andy and Brian both practise tennis. They decide to play 4 matches in the year. The probability that Brian wins is p = 0.4.
The scores are supposed to be independent.
The loser gives the winner 10 £ at the end of each match.

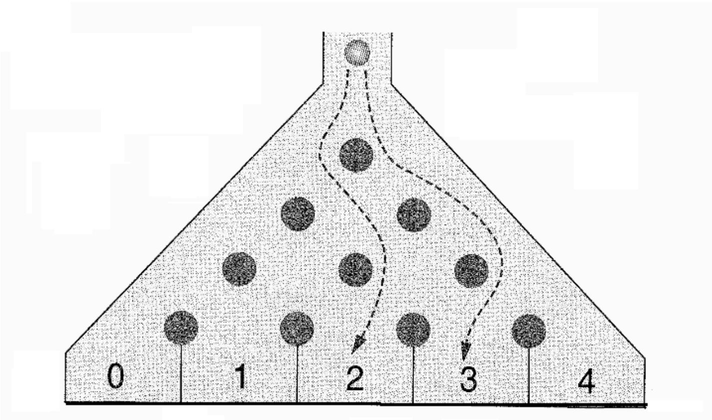
- Find the probability that Brian wins just once.
- Find the probability that Brian wins at least once.
- State the probability distribution associated with Brian’s expense.
- Calculate Brian’s expense expectation at the end of the year.

Task 6 : Write a presentation about Galton and the Bean Machine.

The Galton Board (Bean Machine)



In the following problem, there are 4 rows of pins in our Galton Board.



We wish to determine the probability distribution of the bins at the bottom (0; 1; 2; 3; 4).

I. <u>Preliminary</u>

We consider the event “the ball bounces right”.

- Check that each row corresponds to a Bernoulli trial.

- Give the probability of success, denoted p.

p=

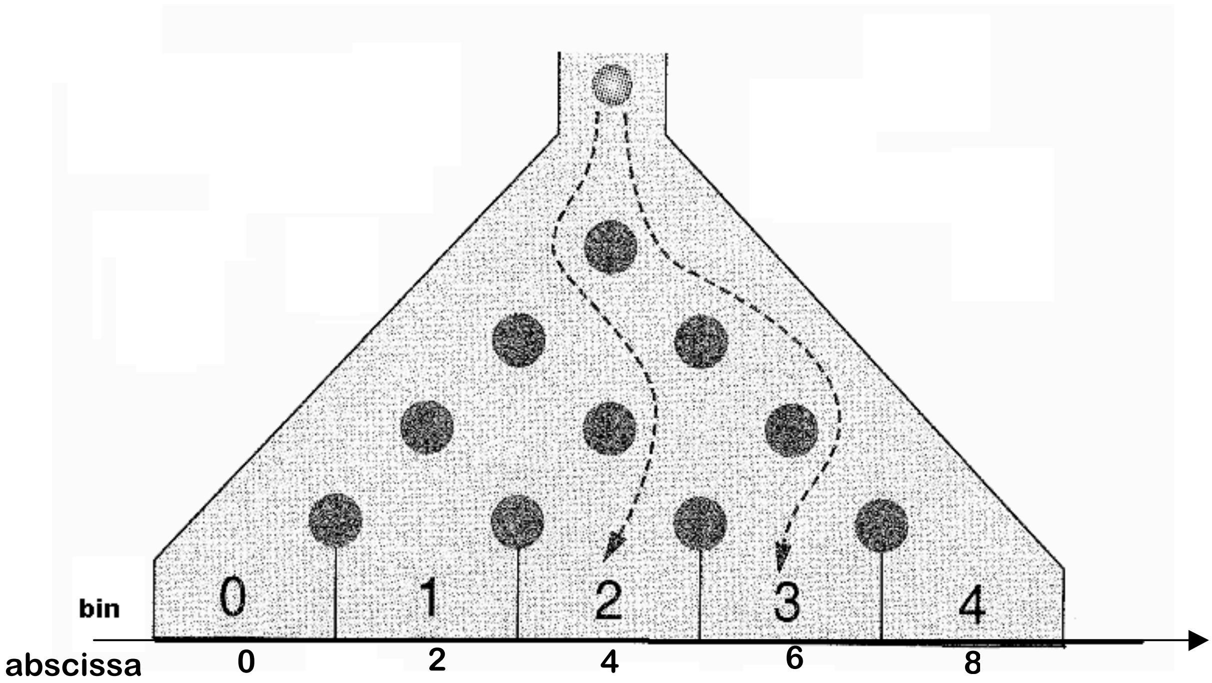
- Draw the Bernoulli tree that describes the game.

- State the probability distribution on {0; 1; 2; 3; 4}.

k	0	1	2	3	4
P(y=k)					

II. Simulation using a spreadsheet

It is convenient to localize the balls by their abscissas.



- *Complete the table:*

Bin	0	1	2	3	4
Abscissa					

- *At the start, the abscissa is …..*

- *Each row :*

  *If the ball bounces right, add **+1** to its abscissa.*
  *If the ball bounces left, add **- 1** to its abscissa.*

  Example: the trajectory of a ball is RRLR. Find the abscissa at the finish.
  At the finish, the abscissa is …..
  Copy the following table on the spreadsheet (you have 200 balls).

	A	B	C	D	E	F	G	H	I	J
1	Ball number	Departure Abscissa	Row 1	Row 2	Row 3	Row 4				Arrival Abscissa
2	1	4								
3	2	4								
4	3	4								
5	4	4								
6	5	4								
7	6	4								
8	7	4								

1. In cell C2, write a formula that gives +1 or – 1 at random.
2. Extend to cell F201.

3. In cell J2, write a formula to find the abscissa of the ball at arrival. Extend to the 200th ball.

4. In the same sheet, construct the table below :

K	L	M	N	O	P	Q
	Arrival Abscissa	**0**	**2**	**4**	**6**	**8**
	Bin number	0	1	2	3	4
	Number of balls					
	Frequency (%)					

5. In cell M3, use the function COUNTIF ( ) to find the number of balls that fell in bin 0 (abscissa 0).
   Extend to bin 4.
6. Find the corresponding relative frequency (%) in cell M4.
7. Draw the bar-chart of the number of balls in terms of the bin.
8. Try again if n = 500 ; n = 1000.
9. Compare with the results of part 1 : Is the frequency close to the probability ?

The shape of the chart is……………………………..or……………………………It seem to represent a ………………………distribution.