Task 14: Why are there exactly Five Regular Polyhedra?

Below is Euclid's proof. Use the tool box to fill in the blanks.

1. Use Polydron® to build solids. What is the minimum number of faces to be connected at a vertex to build a 3D shape?

Each vertex of the solid must be formed by joining at least __________ faces.

2. The sum of the angles formed by the faces at a vertex must be less than _____°.

Explanation (Use Polydron® as a help):

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3. Since the angles at all vertices of all faces of a Platonic solid are identical, and at least __________ faces are joined at a vertex, the size of the angle of each face must be less than _________°.

As regular polygons of six or more sides have only angles of 120° or more, the shape of the face is limited to either a ______________, a _____________, or a __________________.

4. We now determine what is possible with these faces:

- Triangular faces: Each vertex of a regular triangle is 60°, so a solid made of triangles may have _______, _______, or _________ triangles meeting at a vertex; these are the _______________, ____________, and _____________ respectively.

  Note that 6 or more triangles meeting at a vertex gives an angle sum of ______° that is too _________.
- Square faces: Each vertex of a square is 90°, so for such a polyhedron there is only one arrangement possible with _________ faces at a vertex, and it gives the _______________.
- Pentagonal faces: each vertex is 108°; again, only one arrangement, of _________ faces at a vertex is possible, the corresponding polyhedron is the __________________.

three	360	dodecahedron		hexahedron	120	five	three	icosahedron	three
pentagon	large	four	360	tetrahedron	octahedron	triangle	square	three